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RECONSTRUCTION OF A 3-MANIFOLD BY A NON-SINGULAR FLOW

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1. SPINES INDUCED BY A NON-SINGULAR FLOW

Let M be a smooth and closed 3-manifold, and ψ_t be a non-singular flow on M. Take a compact local section Σ of ψ_t so that it is homeomorphic to a compact 2-disk and intersects with every orbit of ψ_t. For such a pair (ψ_t, Σ) we define functions $T_+(x)$ and $T_-(x)$ by

$$T_+(x) = \inf\{ t > 0 \mid \psi_t(x) \in \Sigma \}$$

$$T_-(x) = \sup\{ t < 0 \mid \psi_t(x) \in \Sigma \} .$$

Moreover define $\hat{T}_\pm(x)$ to be $\hat{T}_\pm(x) = \psi_\sigma(x)$ $(\sigma = T_\pm(x))$.
We can take Σ so that it satisfies that

(i) $\partial\Sigma$ is ψ_t-transversal at $(x, T_+(x))$ for any $x \in M$ (see [2] for the definition of ψ_t-transversality), and

(ii) if $x \in \partial\Sigma$ and $x_1 = \hat{T}_+(x) \in \partial\Sigma$, then $\hat{T}_+(x_1)$ is included in $\mathrm{int}(\Sigma)$.

We call a pair (ψ_t, Σ) with the above conditions a <u>normal pair</u>.
For a normal pair (ψ_t, Σ), the <u>flow-spines</u> $P_- = P_-(\psi_t, \Sigma)$ and $P_+ = P_+(\psi_t, \Sigma)$ are defined by

$$P_- = \Sigma \cup \{\psi_t(x) \mid x \in \partial\Sigma \ , \ T_-(x) \leqq t \leqq 0 \}$$

$$P_+ = \Sigma \cup \{\psi_t(x) \mid x \in \partial\Sigma \ , \ 0 \leqq t \leqq T_+(x) \}.$$

Each of P_- and P_+ is a closed fake surface and forms a standard spine of M (cf. [1]). The set $S_j(P_-)$ $(j = 2, 3)$ of the j-th singularities of P_- are given by

$$S_3(P_-) = \{ x \in \mathrm{int}(\Sigma) \mid \hat{T}_+(x) \text{ and } \hat{T}_+^2(x) \text{ are both on } \partial\Sigma \}.$$

$$S_2(P_-) = \hat{T}_-(\partial\Sigma) \cup \{\psi_t(x) \mid x \in S_3(P_-) \ , \ 0 \leqq t \leqq T_+(x)\}$$

(see [1] and [2] for the precise).

2. RECONSTRUCTION OF M.

Let $B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leqq 1\}$ be the unit 3-ball in $\mathbb{R}^3$, $\rho : \partial B \to \partial B$ be a map defined by $\rho(x, y, z) = (x, y, -z)$, and ι be an embedding of Σ into $S^2 = \partial B$ such that $\iota(\partial\Sigma) = \partial B \cap \{z = 0\}$. We define an equivalence relation "$\sim$" on ∂B as follows:

(i) for $x \in S_3(P_-) \subset \mathrm{int}(\Sigma)$,

$$\iota(x) \sim \iota(\hat{T}_+(x)) \sim \iota(\hat{T}_+^2(x)) \sim \iota(\rho(\hat{T}_+^3(x)))$$

(ii) for $x \in \mathrm{int}(\Sigma) \cap (\hat{T}_-(\partial\Sigma) - S_3(P_-))$,

$$\iota(x) \sim \iota(\hat{T}_+(x)) \sim \iota(\rho(\hat{T}_+^2(x)))$$

(iii) for $x \in \mathrm{int}(\Sigma) - \hat{T}_+^2(\partial\Sigma)$,

$$\iota(x) \sim \iota(\rho(\hat{T}_+(x))) \quad .$$

Then we get the following theorem which gives a polyhedral representation of M.

THEOREM 1 ([2]).

M is homeomorphic to $B/\sim$ *, and each of* P_- *and* P_+ *is homeomorphic to* $\partial B/\sim$.

3. AN APPLICATION

We consider a normal pair for which the following condition (B) is satisfied.

(B) $\hat{T}_+(S_3(P_-))$ and $\hat{T}_+^2(S_3(P_-))$ are separated by two points, namely, there are two points z_1 and z_2 on $\partial\Sigma$ such that $\hat{T}_+(S_3(P_-))$ is included in one of the components of $\partial\Sigma - \{z_1, z_2\}$ and $\hat{T}_+^2(S_3(P_-))$ is in the other.

This condition is a generalization of the condition (A) in [4] . And, using the reducing method shown in [3] and [4], we can deform a normal pair with (B) into one giving a polyhedral representation of some normal form which include those given by Fig.7 in [4]. As examples, we exhibit in Fig.1 and 2 the normal form obtained by the above way which represent the lens space $L(5, 1)$ and $L(5, 2)$ respectively. In general, we can prove that the polyhedral representation of these normal form yield S^3 or $S^2 \times S^1$ or lens space $L(p, q)$. Conversely, using the method in [4], we can construct a normal pair with the condition (B) on these manifolds. Thus we have

THEOREM 2.

M admits a normal pair satisfying (B) if and only if $M = S^3$ or $S^2 \times S^1$ or $L(p, q)$.

REFERENCES

[1] Ikeda, H., Acyclic fake surfaces, Topology 10 (1971), 9-36.

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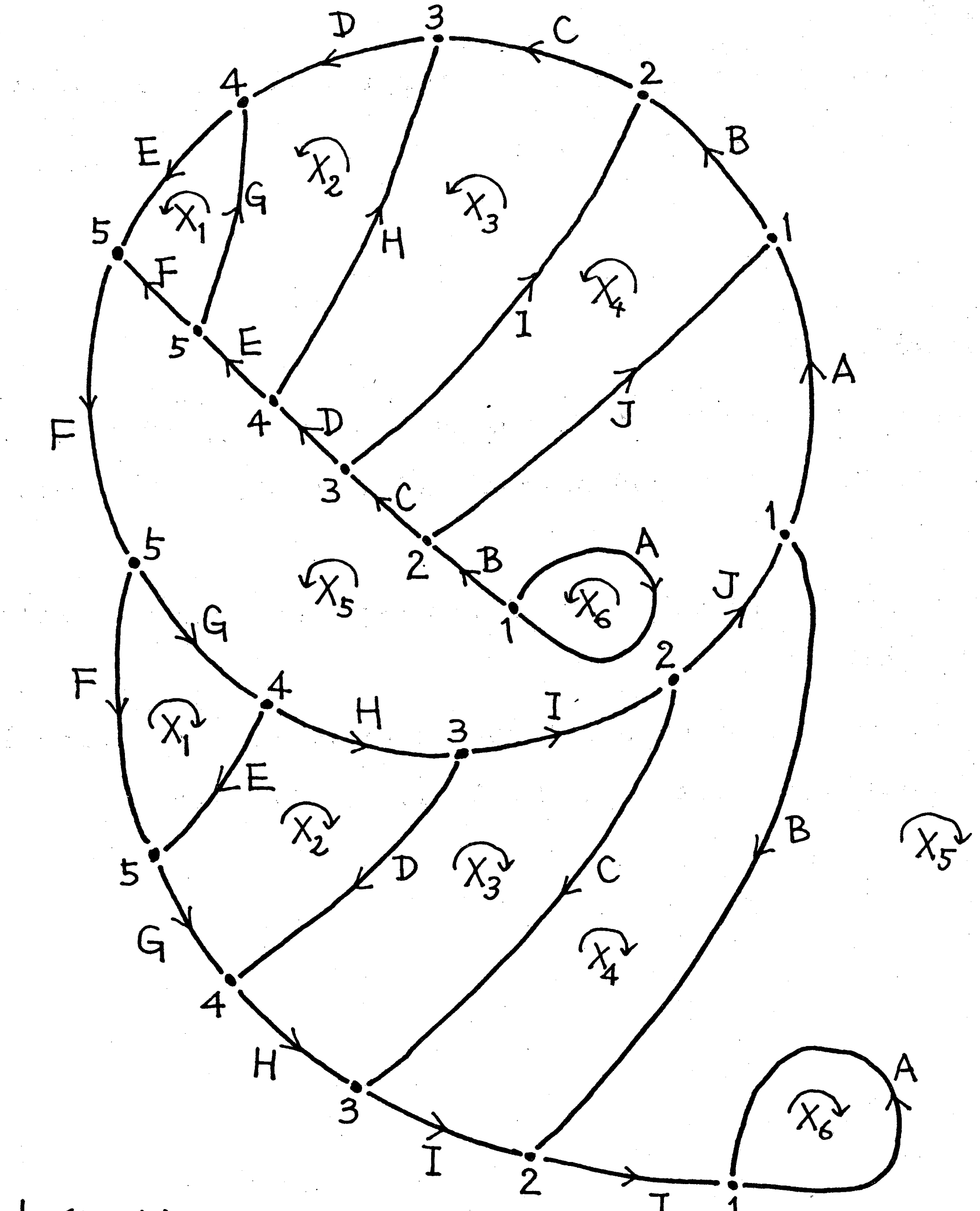
[4] ———, Combinatorial construction of a non-singular flow on a 3-manifold, Kobe J. Math. 3 (1987), 201-208.



L(5, 1)

Fig. 1

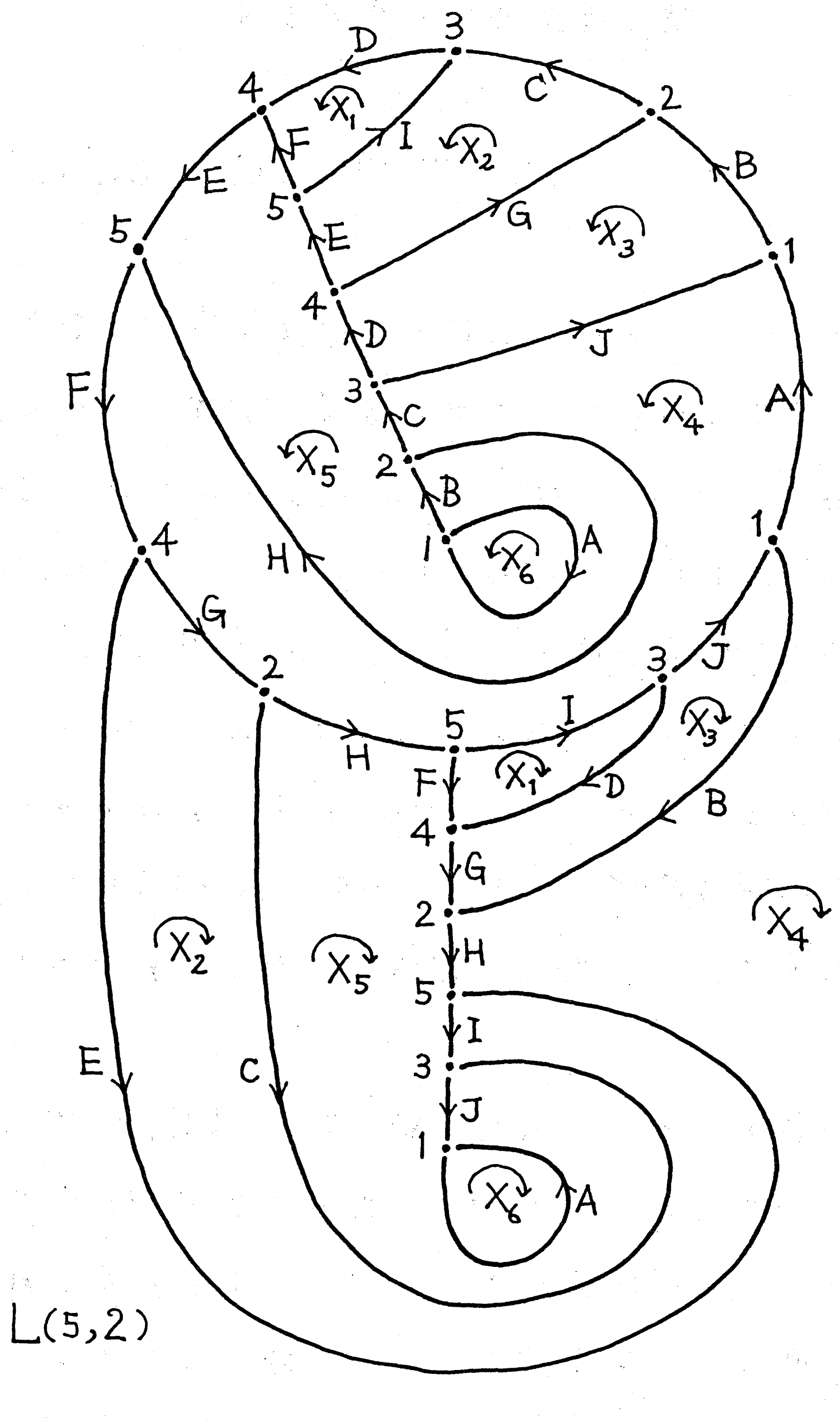


Fig. 2